Tight Bounds for Online Tree Augmentation

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Joint work with Joseph (Seffi) Naor (Technion)
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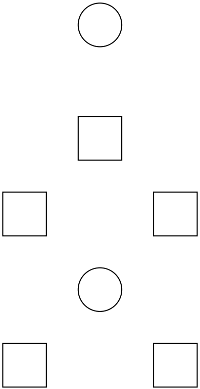
Survivable Network Design Problem

Given an undirected network $G = (V, E)$, costs $c_e \geq 0$ for $e \in E$, source-sink pairs s_1-t_1,..., s_k-t_k, and requirements $r_1, \ldots, r_k$, find minimum-cost edges $F \subseteq E$ such that at least r_i edge-disjoint paths between s_i and t_i for $i = 1, \ldots, k$.

NP-hard even if $r_i = 1$, and $s_i = r$ for all i (Karp '72): Steiner tree problem



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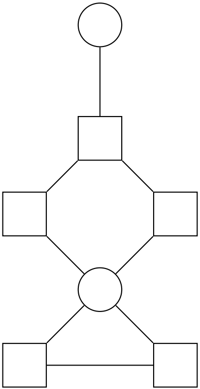

Online Problems

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Quality of algorithm determined by its *competitive ratio*: worst-case ratio over all inputs of cost of algorithm's solution to the minimum-cost solution for all requirements in the input.

Online Steiner Tree

An early case: Imase and Waxman (1991) give an $O(\log k)$-competitive algorithm for the online Steiner tree problem, which $r_i = 1$ and $s_i = r$ for all i. They also show that any algorithm must have competitive ratio at least $\Omega(\log k)$.

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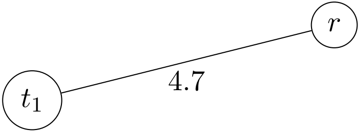

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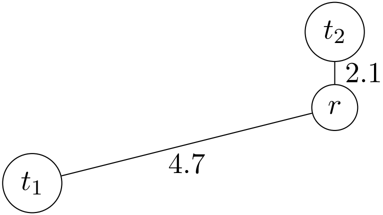

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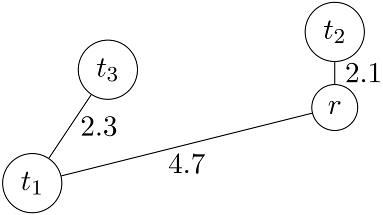


A Quick Analysis

Let c_i be cost algorithm pays to connect t_i when it arrives. Let Z_j be set of indices i with $c_i \in [2^j, 2^{j+1})$.

Algorithm's cost is then

$$\sum_j \sum_{i \in Z_j} c_j \leq \sum_j 2^{j+1} |Z_j|.$$

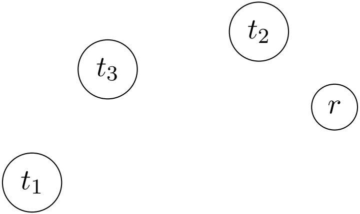
A Quick Analysis

Lemma

For any j, $OPT \geq 2^{j-1}|Z_j|$.

Proof.

Cost of path between any pair of vertices in Z_j is at least 2^j.
Put disjoint balls of radius 2^{j-1} around each point in Z_j. $\square$



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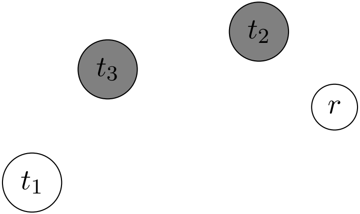

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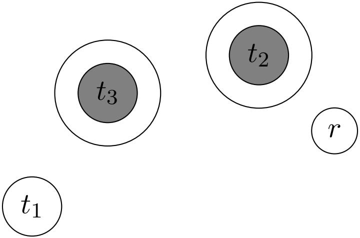

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If ℓ highest index such that $Z_\ell \neq \emptyset$, then:

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\begin{aligned}
2^{\ell+1}|Z_\ell| &\leq 4 \cdot \mathrm{OPT} \\
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Summing the inequalities together, we get that the algorithm's cost is at most $O(\log k)$OPT.

Higher Connectivities

$O(\log k)$-competitive algorithm known for $r_i = 1$, arbitrary s_i-t_i pairs (Berman, Coulston 1997), other types of connectivity (Qian, Umboh, W 2018), node-weighted problems (Hajiaghayi, Liaghat, Panigrahi 2013).

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Question

Can we do better? Better competitive ratio? Deterministic algorithm?

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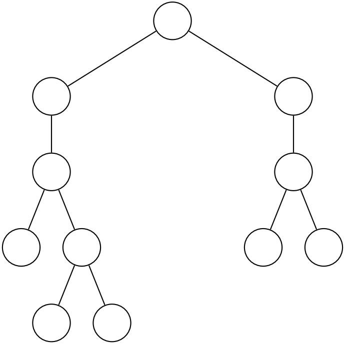

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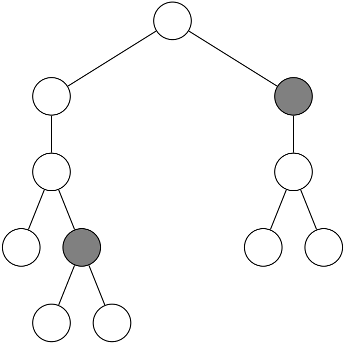

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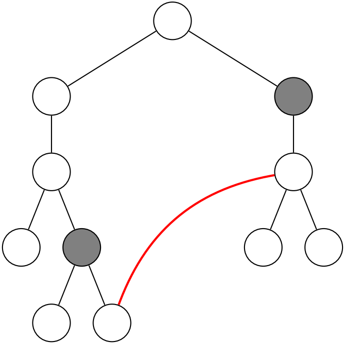

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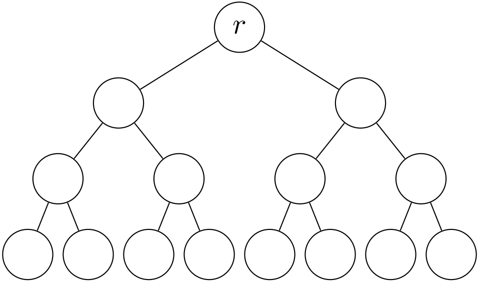

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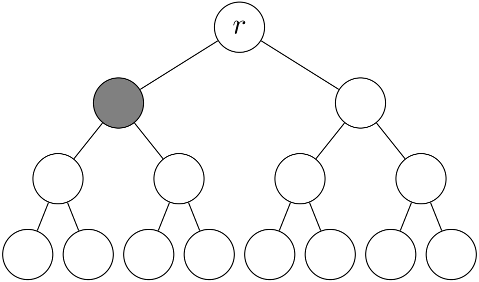

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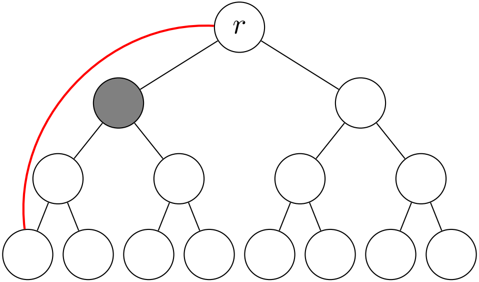

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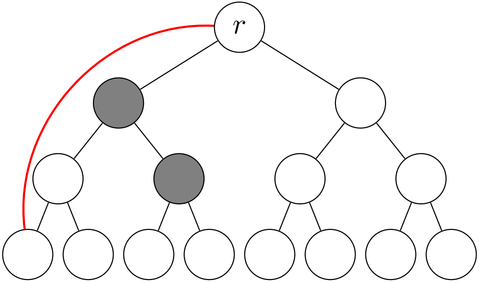

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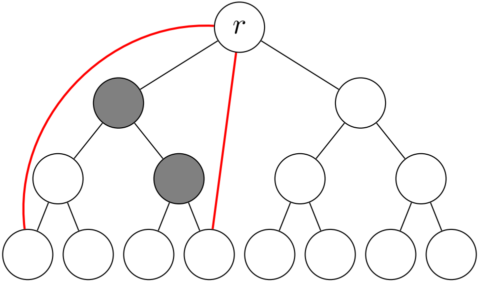

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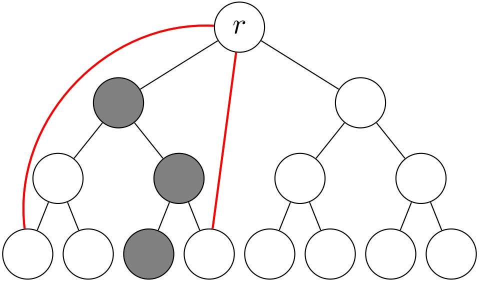

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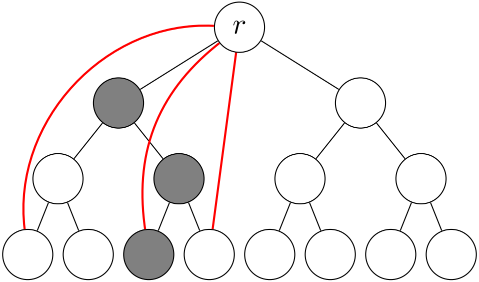

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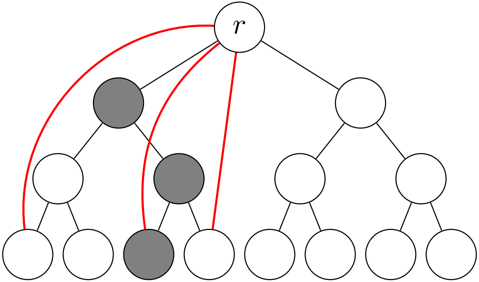

Optimal only buys last link, algorithm must buy $\log_2 n - 1$ links.

Our Result

Theorem (Naor, Umboh, W 2019)

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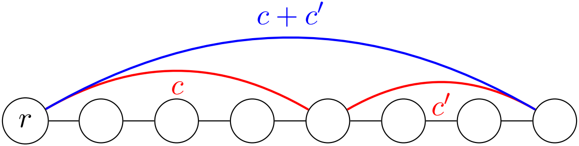
Main ingredients:

1. An algorithm for paths
2. Decomposition of trees into paths
3. A refined path algorithm

Ingredient 1: An Algorithm for Paths

Suppose tree T is a path P, all requests are *rooted*: (r, t_i). Assume WLOG:

- no nonrooted links exist;



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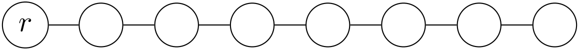

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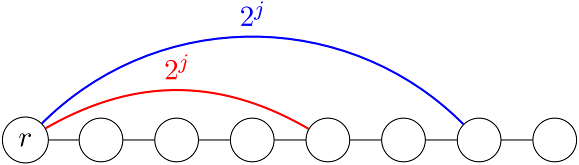

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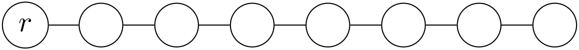

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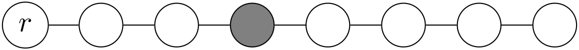

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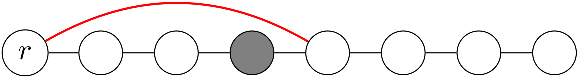

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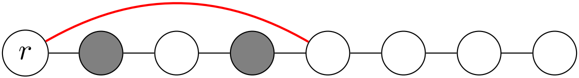

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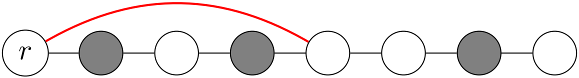

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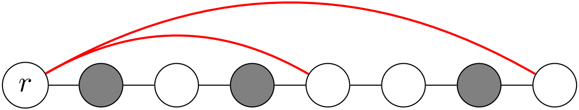

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Theorem

The algorithm is $O(1)$-competitive.

Proof.

Factor of 2 for rounding link costs up to nearest power of 2.

Algorithm buys at most one link of cost 2^j for each j. Consider request (r, t_i) such that cheapest link that covers request is 2^ℓ for ℓ maximum. Then

$$\mathrm{OPT} \geq 2^\ell,$$

while algorithm pays at most

$$2^\ell + 2^{\ell-1} + 2^{\ell-2} + \cdots = 2^{\ell+1} \leq 2 \cdot \mathrm{OPT}.$$

□

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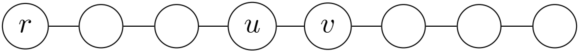

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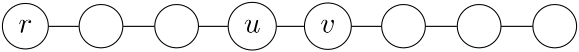

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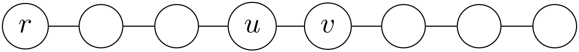

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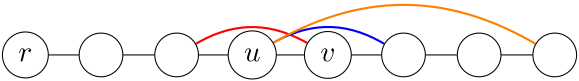

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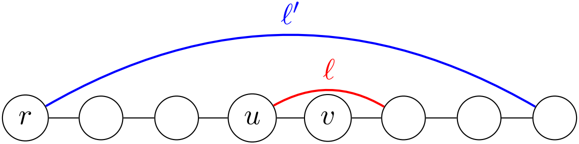

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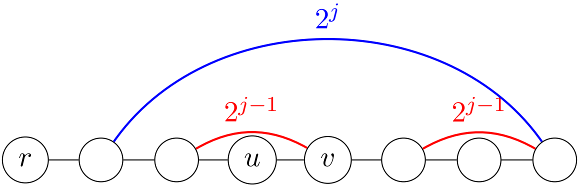
- Any link of cost 2^j contains at most 2^k disjoint links of cost 2^{j-k}.



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Claim

$$OPT \geq \frac{1}{2} \cdot 2^j \cdot |Z_j|.$$

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Algorithm: If request $(u, v) \in P$ not covered, buy (two) cheapest link(s) covering (u, v).

Let Z_j be the set of links of cost 2^j bought by algorithm, so that algorithm's cost is

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Proof.

Essentially the same as for online Steiner tree. □

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Theorem (Naor, Umboh, W 2019)

Any deterministic algorithm for the online path augmentation problem has competitive ratio $\Omega(\log n)$.

Improves on a result of Meyerson (2005) of $\Omega(\log n / \log\log n)$.

Ingredient 2: Tree Decomposition

Theorem (Sleator, Tarjan (1983))

Any rooted tree T can be decomposed into disjoint paths $\mathcal{P}$ such that each path in $\mathcal{P}$ is rooted (has an LCA closest to root), any path in T intersects at most $O(\log n)$ paths in $\mathcal{P}$.

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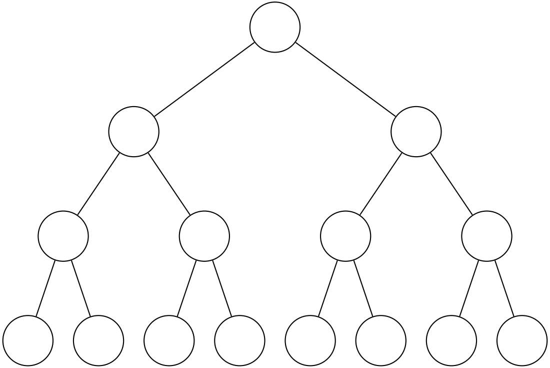

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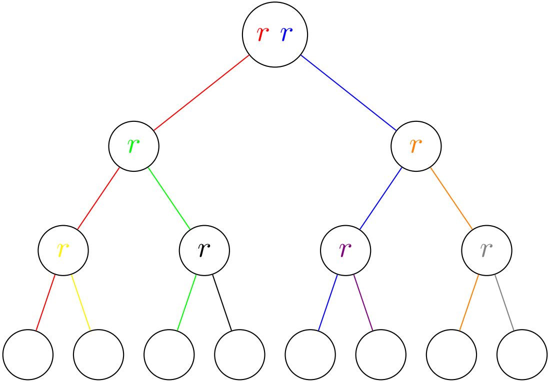

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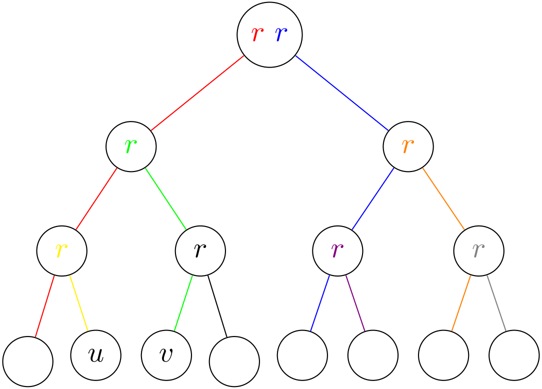
Let $\mathcal{P}$ be a decomposition of tree T into rooted paths.

Definition

The *projection* of a link (u, v) on to a rooted path $P \in \mathcal{P}$ is the link whose endpoints are the endpoints of $P \cap T(u, v)$, where $T(u, v)$ is the u-v path in T.



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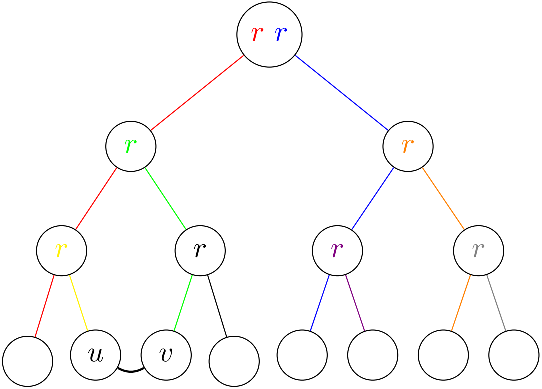

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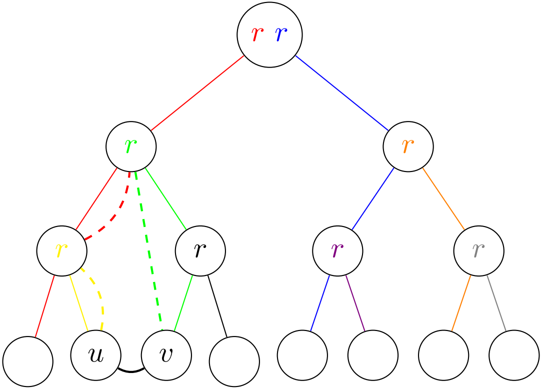

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How can we do better?

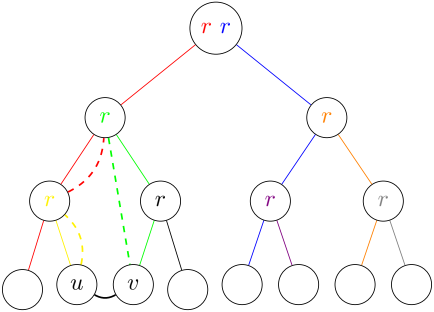
Ingredient 2: Tree Decomposition

Definition

A projection of a link (u, v) on to a rooted path P is *rooted* if one endpoint of the projection is the root of the path P.

Lemma

For any given link (u, v), its projection on to all but one path $P \in \mathcal{P}$ is rooted.



Ingredient 3: Refined Path Algorithm

Definition

An online algorithm for path augmentation is *nice* if for any feasible solution F^* it produces a solution of cost at most

$$O(1)c(R^*) + O(\log n)c(S^*),$$

where R^* are the rooted links in F^* and S^* are the non-rooted links in F^*.

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Theorem

Given a deterministic nice algorithm for online path augmentation, we get a deterministic $O(\log n)$-competitive algorithm for online tree augmentation.

Proof Sketch

Theorem

Given a deterministic nice algorithm for online path augmentation, we get a deterministic $O(\log n)$-competitive algorithm for online tree augmentation.

Proof.

For feasible solution F^* for the tree augmentation problem, let R_P^* be links of F^* whose projections on to $P \in \mathcal{P}$ are rooted, S_P^* be links of F^* that have projections on to P are non-rooted. Then cost of algorithm's solution is at most

$$\sum_{P \in \mathcal{P}} (O(1)c(R_P^*) + O(\log n)c(S_P^*)) \leq O(\log n)c(F^*).$$

$\square$

The Rest

Some amount of work needed to get all of the ideas to work together.

Open Questions

Recall that Gupta, Krishnaswamy, Ravi (2012) give a $O(r_{\max} \log^3 n)$-competitive algorithm for online survivable network design.

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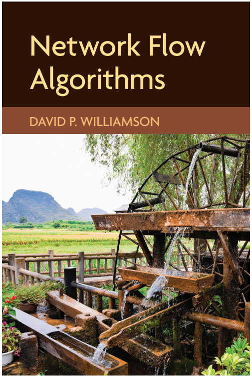
- Is the linear dependence on $r_{\max}$ necessary?
- Are the polylogs necessary?
- Is there an $O(\log n)$-competitive algorithm in the case $r_{\max} = 2$?

Other Work

I also spent the semester finishing a book, to be published by Cambridge this fall.

Online PDF available at
`www.networkflowalgs.com/book.pdf`.



Thanks for your attention.